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**STABILITY OF POLYMER-BASED COMPOSITE CYLINDRICAL THIN-WALLED  
STORAGE TANK REINFORCED WITH CARBON NANOTUBES UNDER  
THERMAL FIELD**

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**ABSTRACT**

In this research polymer (polyethylene) thin-walled cylindrical membrane reinforced with carbon nanotubes fibers are studied under linear, axial and uniform temperature load. The stress and strains are obtained using mechanical theories of composite materials and then one obtained the whole energy of storage tank including total strain energy and work from temperature axial distribution. Buckling modes have been assumed as sinusoidal and cosine polynomials and simple support conditions are considered at two ends of storage tank. Buckling modes are placed in stability equations and using minimum potential energy principle, the critical temperature for storage tank is calculated and depicted in terms of buckling modes. For analytical solution one used Daniel nonlinear relationship and composite membrane classical theory. In this research van der Waals forces have been examined in interface with using method of Leonard Jones. Furthermore one has examined the impact of change in angle of orientation of fiber with respect to cylinder axis, effect of increase of volume percentage of fibers inside the polymer, effect of change in length to radius ratio and change in membrane thickness on stability of composite membrane.

**Keywords: Thermal buckling, nanocomposite, critical thermal degree, carbon nanopipe, composite cylindrical membrane**

**INTRODUCTION**

Buckling is one of types of instability that emerges in structural members under

pressure. In other word this phenomenon emphasizes on physical behavior difference

of a thin member under compression and tension and from practical point of view the buckling critical load is considered as one of the most important issues in engineering design. Buckling or creasing usually happen suddenly in membranes and sheets and are extremely dangerous. Therefore it is necessary to evaluate these phenomenon for preventing their occurrence and restricting the damages in case of their occurrence.

Thin-walled storage tank under pressure and heat has abundant applications in various industries such as petroleum, aerospace and so on. Composite thin-walled tanks due to high strength to weight ratio has been vastly used within recent decades. Carbon nanotubes due to their higher strength than strongest steels and extremely light weight are appropriate option for increasing thermal stability in composite structure.

### Potential energy for an elastic object

$$V = U - \int_{S_r} T_i u_i dS - \int_V F_i u_i dV \quad \text{Eq. 1}$$

In above relationship the second and third relationship are works done by surface tension and body forces respectively body forces are forces depending to object mass and because here homogenous bodies are used, therefore the body forces are proportionate to the volume. U is elastic object strain energy that includes matrix

The above relationship can be rewritten as follows.

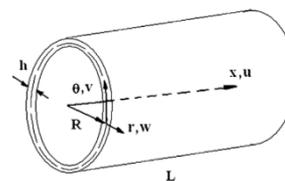
strain energy and fibers as well as cohesion energy in interface of carbon nanotube and matrix.

### Minimum potential energy principle

Among all displacements that satisfies known border conditions and compatibility, those that satisfy equilibrium condition minimize the potential energy, therefore in an approximated solution the compatibility conditions are satisfied completely and equilibrium condition are satisfied approximately.

For using this principle one can assume that changes in potential energy is zero, it means:

$$\Delta V = 0 \quad \text{Eq.2}$$



**Figure 1- geometry of thin-walled cylinder membrane Calculation of strain energy of composite membrane based on cohesion in interface**

When we use separation assumption in interface of nanotube and polymer, strain energy of membrane would be to some extent different from other composite substances. in these conditions the strain energy is written as following equation.

$$U = \frac{1}{2} \int_V \sigma \varepsilon dV + \int_{S_{int}} \phi dA$$

$$\text{Eq.3}$$

$$U = \frac{1}{2} \int_V \sigma \varepsilon dV + \int_{S_{int}} \left\{ \phi - \frac{1}{4} \sigma^{int} ([u] \otimes n + n \otimes [u]) \right\} dA \quad \text{Eq.4}$$

By assuming that the displacement of void space is perpendicular to carbon nanotube surface, the above relation can be simplified as follows.

$$U = \frac{1}{2} \int_V \sigma \varepsilon dV + \int_{S_{int}} \phi dA - \frac{1}{2} \int_{S_{int}} \sigma^{int} (2[u]) dA \quad \text{Eq. 5}$$

The above relationship can be considered as follows:

$$\begin{aligned} U &= U_e + U_{cohesive} - U_{int} \\ U_e &= \frac{1}{2} \int_V \sigma \varepsilon dV \\ U_{cohesive} &= \int_{S_{int}} \phi dA \\ U_{int} &= \frac{1}{2} \int_{S_{int}} \sigma^{int} \varepsilon^{int} dA \end{aligned} \quad \text{Eq.6}$$

In above relationship  $U_C$  represents the strain energy in nanotube and polymer and  $U_{int}$  represents subtracted energy arising from existing voids among nanotube and polymer. Also in this relationship  $U_{cohesive}$  represents cohesive energy at interface of nanotube and polymer that in fact is arising from van der Waals force between carbon atoms in the interface of nanotube and polymer.

At above relationship  $\sigma$  and  $\varepsilon$  are equivalent composite stress and strain and integration of membrane volume is carried out.  $\sigma^{int}$  and  $\varepsilon^{int}$  are stress and strain in interface of nanotube and polymer and

integration is done on interface of all nanotubes and polymers.

Van der Waals energy between two atoms at the distance of ( $r$ ) is stated as follows:

$$V(r) = 4\varepsilon \left( \frac{\sigma^{12}}{r^{12}} - \frac{\sigma^6}{r^6} \right) \quad \text{Eq. 7}$$

That  $\sqrt[6]{2}\sigma$  is balance distance between atoms and  $\varepsilon$  is intermolecular energy within balance distance that its value is  $\varepsilon = 0.004656 \text{ eV}$  and  $\sigma = 0.3825 \text{ nm}$  for carbon atom from carbon nanotube and unit of  $-\text{CH}_2-$  from polyethylene.

According to Lennard Jones potential function the cohesion law is written as follows [1].

$$\sigma_{int} = 3.07 \sigma_{max} \left\{ \frac{1}{\left[ 1 + 0.682 \frac{\sigma_{max} u}{\phi_{total}} \right]^4} - \frac{1}{\left[ 1 + 0.682 \frac{\sigma_{max} u}{\phi_{total}} \right]^{10}} \right\} \quad \text{Eq. 8}$$

Where  $\sigma_{max} = \frac{6\pi}{5} \rho_p \rho_c \varepsilon \sigma^2$  and  $\phi_{total} = \frac{4\pi}{9} \sqrt{\frac{5}{2}} \rho_p \rho_c \varepsilon \sigma^3$ . In above equation  $u$  is in meter and  $\sigma_{int}$  is in Pa.

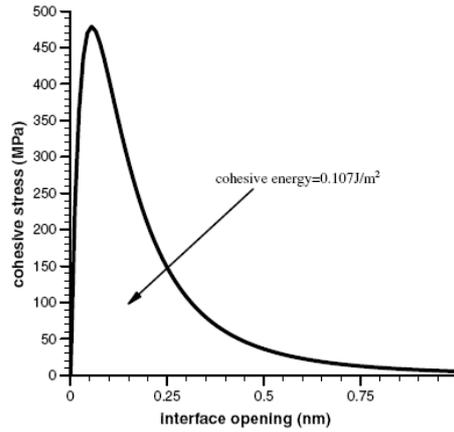


Figure 2 shows changes in amount of cohesive stress of interface versus interface opening. As it is shown in the figure, the cohesion law in interface has three stages. 1- Stiffening 2- softening 3- complete separation.

The cohesion relationship in first part (linear) is stated as follows:

$$\sigma^{int} = K_{\sigma} u$$

$$K_{\sigma} = 12.56244 \frac{\sigma_m^2}{\phi_{total}} \quad (9)$$

Surface cohesion energy under  $\sigma^{int}$  is versus opening displacement, therefore total cohesive energy is calculated from following relationship.

$$\phi = \frac{K_{\sigma} u^2}{2} (j/m^2) \quad (10)$$

In fact  $\phi$  represents cohesive energy per area unit of interface of nanotube and polymer. Form other side by considering an element from membrane with volume of  $dsdx dz$ , the carbon nanotube volume in this element would be  $f dsdx dz$  that  $f$  represents fibers volume percentage. By considering

$$d\phi = \frac{K_{\sigma} K_n^2 f \bar{E}^{-2} \bar{\epsilon}^{-2}}{r} dsdx dz, ds = Rd\theta \quad (11)$$

Therefore the surface integral of cohesive energy relation turns into volume integral as follows.

volume to lateral area ratio of nanotube all interfaces of nanotube and polymer in this element of membrane would be  $2f dsdx dz/r$ . Then cohesive energy in a volume element of membrane can be as follow.

$$d\phi = \frac{K_{\sigma} f u^2}{r} dsdx dz, ds = Rd\theta \quad (12)$$

The relation between opening and macroscopic strain is specified as follows.

$$u = K_n \bar{E} \bar{\epsilon} \quad (13)$$

By putting Eq. 12 in Eq. 11 cohesive energy for volume element of membrane can be obtained as follows.

$$U_{cohesive} = \int_{S_{int}} \phi dA = \int_V \frac{K_\sigma K_n^2 f E^{-2} \varepsilon^{-2}}{r} dV \quad \text{Eq. 14.}$$

The number of nanotubes in a volume elements can be calculated by dividing volume of whole of nanotubes in membrane element ( $f ds dx dz$ ) on volume of one nanotube ( $\pi r^2 l$ ).

$$n_{nanotube} = f ds dx dz / \pi r^2 l \quad \text{Eq.15}$$

In this research one has ignored sliding between nanotube and polymer.

With place values of  $\sigma^{int}$  and  $\varepsilon^{int}$  in terms of  $u$  in fourth part of Eq. 6 as well as considering the number of nanotubes, the following relationship can be obtained.

$$\frac{U_{int}}{V_{shell}} = \left[ \frac{1}{2} \int_0^u (K_\sigma u) \left( \frac{2u}{3r} \right) 2\pi r l du \right] n_{nanotube} \quad \text{Eq. 16}$$

After integration and placing number of nanotubes, the above relationship can be simplified as follows.

$$\frac{U_{int}}{V_{shell}} = \left[ 2K_\sigma u^3 l / 9 \right] \frac{f dV}{\pi r^2 l} \quad \text{Eq.17}$$

Considering above relationship, the opening strain energy in the entire of membrane is obtained as follows.

$$U_{int} = \int_V \frac{2K_\sigma u^3 f}{9r^2} dV \quad \text{Eq. 18.}$$

With placing above relationship and using Eq. 12 the subtracted energy arising from existing opening in voids between nanotubes and polymer is calculated as follows.

$$U = \frac{1}{2} \int_V \sigma \varepsilon dV - \int_V \frac{2K_\sigma K_n^3 f E^{-3} \varepsilon^{-3}}{9r^2} dV + \int_V \frac{K_\sigma K_n^2 f E^{-2} \varepsilon^{-2}}{r} dV \quad \text{Eq. 19.}$$

By ignoring high order terms, the total strain energy is simplified as follows.

$$U = \frac{1}{2} \int_V \sigma \varepsilon dV + \int_V \frac{K_\sigma K_n^2 f E^{-2} \varepsilon^{-2}}{r} dV \quad \text{Eq. 20.}$$

Relation between stress and strain for a composite membrane in general state is as follow.

$$\begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_z \\ \sigma_{\theta z} \\ \sigma_{xz} \\ \sigma_{x\theta} \end{bmatrix} = \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} & 0 & 0 & \overline{C}_{16} \\ \overline{C}_{21} & \overline{C}_{22} & \overline{C}_{23} & 0 & 0 & \overline{C}_{26} \\ \overline{C}_{31} & \overline{C}_{32} & \overline{C}_{33} & 0 & 0 & \overline{C}_{36} \\ 0 & 0 & 0 & \overline{C}_{44} & \overline{C}_{45} & 0 \\ 0 & 0 & 0 & \overline{C}_{54} & \overline{C}_{55} & 0 \\ \overline{C}_{16} & \overline{C}_{26} & \overline{C}_{36} & 0 & 0 & \overline{C}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x - \alpha_x T \\ \varepsilon_\theta - \alpha_\theta T \\ \varepsilon_z - \alpha_z T \\ \gamma_{\theta z} - \alpha_{\theta z} T \\ \gamma_{xz} - \alpha_{xz} T \\ \gamma_{x\theta} - \alpha_{x\theta} T \end{bmatrix} \quad \text{Eq. 21.}$$

Where  $\overline{C}_{ij}$  are transformed elastic coefficients in reference coordinate system ( $x, r, \theta$ ).

$$C_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, C_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \quad \text{Eq. 22.}$$

$$C_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}}, C_{66} = G_{12}$$

Where  $E_1$ ,  $E_2$ ,  $\nu_{12}$ ,  $\nu_{21}$  and  $G_{12}$  are respectively composite modules in longitudinal and transversal direction, composite Poisson coefficient in longitudinal and transversal direction and composite shear modules.

$$\begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{bmatrix} = \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & 0 \\ \overline{C}_{21} & \overline{C}_{22} & 0 \\ 0 & 0 & 2\overline{C}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x - \alpha_x T \\ \varepsilon_\theta - \alpha_\theta T \\ \varepsilon_{x\theta} - \alpha_{x\theta} T \end{bmatrix} \quad \text{Eq. 23}$$

Therefore Eq. 20 of matrix form can be rewritten as:

$$U = \frac{1}{2} \int_V [\sigma][\varepsilon] dV + \int_V \frac{K_\sigma K_n^2 f \bar{E}^{-2} \varepsilon^2}{r} dV \quad \text{Eq. 24.}$$

The above relationship can be rewritten as follows.

$$U = \frac{1}{2} \int_V [\varepsilon_x - \alpha_x T \quad \varepsilon_\theta - \alpha_\theta T \quad \varepsilon_{x\theta} - \alpha_{x\theta} T]^T \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & 0 \\ \overline{C}_{21} & \overline{C}_{22} & 0 \\ 0 & 0 & 2\overline{C}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x - \alpha_x T \\ \varepsilon_\theta - \alpha_\theta T \\ \varepsilon_{x\theta} - \alpha_{x\theta} T \end{bmatrix} dV + \int_V \frac{K_\sigma K_n^2 f \bar{E}^{-2} \varepsilon^2}{r} dV \quad \text{Eq. 25.}$$

### Strain-displacement relationship

The displacement field in shear theory of cylindrical membranes third order by ignoring high order terms is presented as follows.

$$\begin{cases} u = u_0 + z\beta_x & \beta_x = -\frac{\partial w}{\partial x} \\ v = \frac{w}{R} + z\beta_\theta & \beta_\theta = -\frac{\partial w}{\partial \theta} \\ w = w_0(x, \theta) \end{cases} \quad \text{Eq. 26}$$

According to classic theory assumptions, displacement of  $u$ ,  $v$  and  $w$ , in any point with respect to distance  $z$  in direction of membrane radius are obtained as follow.

$$\begin{cases} \varepsilon_x = \frac{\partial u_0}{\partial x} + z \frac{\partial \beta_x}{\partial x} \\ \varepsilon_\theta = \frac{w}{R} + \frac{z}{R} \frac{\partial \beta_\theta}{\partial \theta} + \frac{1}{R} \frac{\partial v_0}{\partial \theta} \\ \varepsilon_{x\theta} = \frac{1}{2} \left( z \frac{\partial \beta_\theta}{\partial x} \right) + \frac{\partial v_0}{\partial x} + \frac{1}{R} \frac{\partial u_0}{\partial \theta} + \frac{z}{R} \frac{\partial \beta_x}{\partial \theta} \\ \varepsilon_{x\theta} = \frac{1}{2} \left( \beta_x + \frac{1}{R} \frac{\partial w}{\partial x} \right) \\ \varepsilon_{\theta z} = \frac{1}{2} \left( \beta_\theta + \frac{1}{R} \frac{\partial w}{\partial \theta} \right) \end{cases} \quad \text{Eq. 27.}$$

According to these relationships between strain-displacement and by considering the symmetry, membrane geometry is obtained as follows.

Considering membrane classic theory and by ignoring shear deformation,  $\varepsilon_{xz} = \varepsilon_{\theta z} = 0$  and considering the fact that the composite membrane is orthotropic then  $\overline{C}_{16} = \overline{C}_{26} = 0$ . Therefore above relationship can be simplified as follows.

$$\begin{cases} \varepsilon_x = -z \frac{\partial^2 w}{\partial x^2} \\ \varepsilon_\theta = \frac{w}{R} - \frac{z}{R} \frac{\partial^2 w}{\partial \theta^2} \\ \varepsilon_{x\theta} = -\frac{1}{2} \frac{\partial^2 w}{\partial x \partial \theta} z - \frac{z}{2R} \frac{\partial^2 w}{\partial x \partial \theta} \end{cases} \quad \text{Eq. 28}$$

### Placing stresses and strains in energy equation

With placing Eq. 28 in Eq.25 and considering the fact that in classic theory at

buckling loads the displacement of middle plane is considered as zero ( $u_0 = v_0 = 0$ ). Therefore total strain energy is obtained from following relationship.

$$U = \frac{1}{2} \int_V \begin{bmatrix} -z \frac{\partial^2 w}{\partial x^2} & \frac{w}{R} - \frac{z}{R} \frac{\partial^2 w}{\partial \theta^2} & -\frac{1}{2} \frac{\partial^2 w}{\partial x \partial \theta} z - \frac{z}{2R} \frac{\partial^2 w}{\partial x \partial \theta} \end{bmatrix} \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & 0 \\ \bar{C}_{21} & \bar{C}_{22} & 0 \\ 0 & 0 & 2\bar{C}_{66} \end{bmatrix} dV + \int_V \frac{k_\sigma k_x^2 f \bar{E}^{-2}}{r} dv \quad \text{Eq. 29}$$

With broadening the above equation the total stain energy is calculated as follows.

$$\begin{aligned} U = & \frac{1}{2} \int_V \bar{C}_{11} \left( z \frac{\partial^2 w}{\partial x^2} \right)^2 + \bar{C}_{12} \left( \frac{w}{R} - \frac{z}{R} \frac{\partial^2 w}{\partial \theta^2} \right) \left( -z \frac{\partial^2 w}{\partial x^2} \right) + \\ & \bar{C}_{12} \left( \frac{w}{R} - \frac{z}{R} \frac{\partial^2 w}{\partial \theta^2} \right) \left( -z \frac{\partial^2 w}{\partial x^2} \right) + \bar{C}_{22} \left( \frac{w}{R} - \frac{z}{R} \frac{\partial^2 w}{\partial \theta^2} \right)^2 + \\ & \bar{C}_{66} \left( \frac{z}{2} \frac{\partial^2 w}{\partial x \partial \theta} + \frac{z}{2R} \frac{\partial^2 w}{\partial x \partial \theta} \right)^2 dv + \int_V \frac{k_\sigma k_x^2 f \bar{E}^{-2}}{r} dv \end{aligned} \quad \text{Eq. 30}$$

Total done work by thermal loads in Eq. 1 is obtained as follows.

$$\int_{s_r} T_i u_i ds = \int_s \frac{1}{2} \left\{ (\bar{C}_{11} \alpha_x + \bar{C}_{12} \alpha_y) \Delta Th \left( \frac{\partial w}{\partial x} \right)^2 \right\} ds \quad \text{Eq. 31}$$

Thus, potential energy is calculated like this:

$$\begin{aligned} V = & \frac{1}{2} \int_V \left\{ (\bar{C}_{11} + D) \left( \frac{\partial^2 w}{\partial x^2} \right)^2 z^2 - 2\bar{C}_{12} \left( z \frac{w}{R} \frac{\partial^2 w}{\partial x^2} \right) + 2\bar{C}_{12} \left( \frac{\partial^2 w}{\partial \theta^2} \right) \left( \frac{\partial^2 w}{\partial x^2} \right) \frac{z^2}{R} + \right. \\ & \left. + (\bar{C}_{22} + D) \left( \left( \frac{w}{R} \right)^2 + \frac{z^2}{R^2} \left( \frac{\partial^2 w}{\partial \theta^2} \right)^2 - 2 \frac{wz}{R^2} \left( \frac{\partial^2 w}{\partial \theta^2} \right) \right) + \right. \\ & \left. \bar{C}_{66} \left( \frac{z^2}{4} \left( \frac{\partial^2 w}{\partial x \partial \theta} \right)^2 + \frac{z^2}{4R^2} \left( \frac{\partial^2 w}{\partial x \partial \theta} \right)^2 + \frac{z^2}{2R} \left( \frac{\partial^2 w}{\partial x \partial \theta} \right)^2 \right) \right\} dv - \\ & - \int_s \frac{1}{2} \left\{ (\bar{C}_{11} \alpha_x + \bar{C}_{12} \alpha_y) \Delta Th \left( \frac{\partial w}{\partial x} \right)^2 \right\} ds \end{aligned} \quad \text{Eq. 32}$$

D parameter is defined as follows:

$$D = \int_V \frac{k_\sigma k_x^2 f \bar{E}^{-2}}{r} dv \quad \text{Eq. 33}$$

### Border conditions of simple support

Considering border conditions of simple support at two ends of membrane, the displacement function is considered as following polynomials.

$$w = 0 \quad \text{at } x = 0, L \quad \text{Eq. 34.}$$

$$\frac{\partial^2 w}{\partial x^2} = 0 \quad \text{at } x = 0, L$$

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin(\alpha_m x) \cos(n\theta), \quad \alpha_m = \frac{m\pi x}{L}$$

With placing the Eq. 34 in Eq. 32 and integrating it, the membrane potential energy with simple support is presented like this:

$$\begin{aligned} V = & \frac{1}{2} \left\{ \int_0^L \int_0^l \left[ \frac{1}{3} (\bar{C}_{11} + D) ((R+h)^3 - R^3) \alpha_m^4 \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 + \right. \right. \\ & \bar{C}_{12} \frac{1}{R} ((R+h)^2 - R^2) \alpha_m^2 \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 + \\ & + \bar{C}_{12} \frac{2}{3R} ((R+h)^3 - R^3) \alpha_m^2 n^2 \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 \\ & + (\bar{C}_{22} + D) \left[ \left( \frac{1}{R} \right)^2 \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 - \right. \\ & \left. \frac{n^4}{3R^2} ((R+h)^3 - R^3) \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 + \right. \\ & \left. \frac{1}{R^2} n^2 ((R+h)^2 - R^2) \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 \right] + \\ & \bar{C}_{66} \left\{ \frac{1}{12} \alpha_m^2 n^2 ((R+h)^3 - R^3) \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 + \right. \\ & \frac{1}{12R^2} \alpha_m^2 n^2 ((R+h)^3 - R^3) \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 + \\ & \left. + \frac{1}{6R} \alpha_m^2 n^2 ((R+h)^3 - R^3) \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 \right\} R dx d\theta \\ & - \int_0^L \int_0^l \frac{1}{2} \left\{ (\bar{C}_{11} \alpha_x + \bar{C}_{12} \alpha_y) \Delta T h \alpha_m^2 \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha_m x \cos n \theta \right)^2 \right\} R dx d\theta \end{aligned} \quad \text{Eq. 35}$$

Now by placing  $\delta V = 0$  and derivation of above relationship in terms of unknowns, the critical temperature of membrane can be obtained as follows.

$$\begin{aligned} \Delta T = & \frac{1}{(\bar{C}_{11} \alpha_x + \bar{C}_{12} \alpha_y) h \alpha_m^2} \left\{ \frac{R}{3} (\bar{C}_{11} + D) ((R+h)^3 - R^3) \alpha_m^4 + \bar{C}_{12} \frac{1}{R} ((R+h)^2 - R^2) \alpha_m^2 + \right. \\ & + \bar{C}_{12} \frac{2}{3} ((R+h)^3 - R^3) \alpha_m^2 n^2 + (\bar{C}_{22} + D) \left[ \frac{1}{R} - \frac{n^4}{3R} ((R+h)^3 - R^3) + \right. \\ & \left. \frac{1}{R} n^2 ((R+h)^2 - R^2) \right] + \bar{C}_{66} \left[ \frac{R}{12} \alpha_m^2 n^2 ((R+h)^3 - R^3) + \right. \\ & \left. \frac{1}{12R} \alpha_m^2 n^2 ((R+h)^3 - R^3) + \frac{1}{6} \alpha_m^2 n^2 ((R+h)^3 - R^3) \right] \end{aligned} \quad \text{Eq. 36.}$$

In above relationship  $m$  and  $n$  are longitudinal half wave number and environmental full wave number respectively.

For obtaining bucking critical load one should examine that in which environmental full wave or longitudinal half wave the buckling takes place. The above equation is ascending in terms of

parameter  $n$ . Therefore the buckling takes place in the first environmental full wave. For specifying buckling longitudinal half wave, one derives from above relationship in terms of  $m$  and makes it equal to zero to obtain the buckling longitudinal half wave. With putting value  $m$  and  $n$  corresponding to buckling load in Eq. 36 the buckling critical temperature can be obtained.

## RESULTS

In this section one examines effect of angle of fibers, fibers volume percentage, carbon nanotubes radius, and length to different radiuses ratio on stability of cylindrical membrane.

### 1- impact of fiber angle in stability of nanocomposite membrane

As it is shown in figure 4, with increase of fibers angle with respect to cylinder axis from  $0$  to  $45^\circ$  the stability of nanocomposite membrane increase and then it decreases. Due to high value of

elastic coefficient in carbon nanotube with respect to the base, fibers volume percentage change has no such effect on fibers optimal angle but this change increases significantly the stability amount.

### 2- impact of length changes in terms of radius in stability of nanocomposite membrane with simple support

It is obvious from figures 5, 6 and 7 that in a certain volume percentage with increase of length to radius ratio at first the buckling critical temperature increases. With increase of carbon nanotube radius (change in type of carbon nanotube) length to radius ratio ( $L/R$ ) corresponding to critical temperature becomes minimal and the buckling critical temperature amount (membrane stability) decreases. Which its reason is reduction of number of nanotube in volume unit in equal volume percentage.

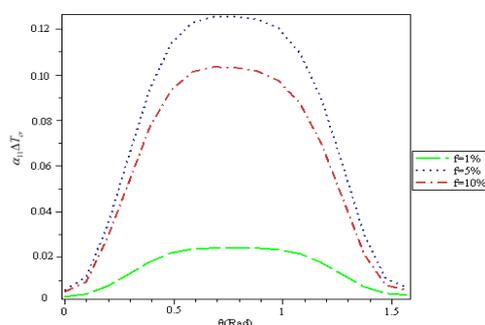


Figure4- changes of  $\alpha_{11} \Delta T_{cr}$  in terms of changes in angle of fibers orientation for Armchair 96,6) with different volume percentage and simple support

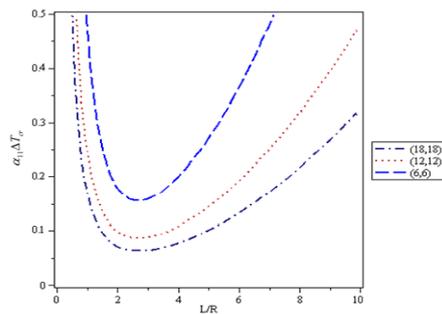


Figure 5- changes of  $\alpha_{11}\Delta T_{cr}$  in terms of L/R change for membrane with fibers of armchair type (18, 18), (12, 12) and (6, 6) with 10 volume percent and fiber angle of  $0^\circ$ , simple support

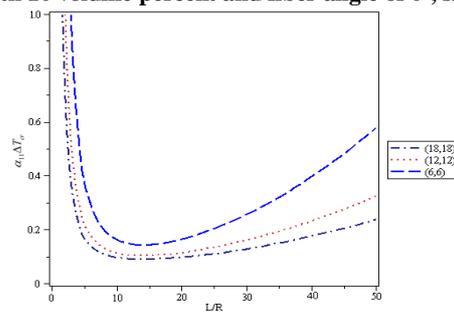


Figure 6. Changes of  $\alpha_{11}\Delta T_{cr}$  in terms of L/R change for membrane with fibers of armchair type (18, 18), (12, 12) and (6, 6) with 10 volume percent and fiber angle, simple support

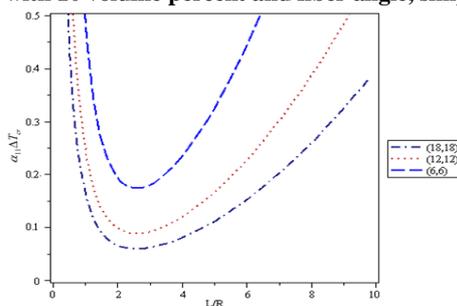


Figure 7. Changes of  $\alpha_{11}\Delta T_{cr}$  in terms of L/R change for membrane with fibers of armchair type (18, 18), (12, 12) and (6, 6) with 10 volume percent and fiber angle of  $90^\circ$ , simple support

It is observable from figures 8, 9 and 10 that at constant volume percentage for certain value of critical temperature, by increase in longitudinal half wave number, the buckling takes place in a greater length to radius ratio.

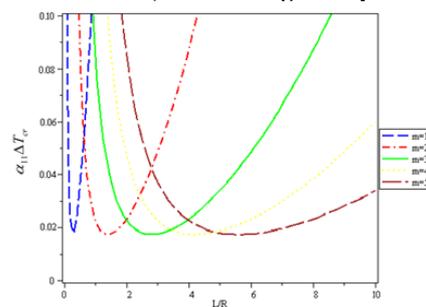


Figure 8. Changes of  $\alpha_{11}\Delta T_{cr}$  in terms L/R for  $f=10\%$  and fibers from type of armchair (6,6) for different longitudinal half waves and fiber angle of  $0^\circ$  and simple support

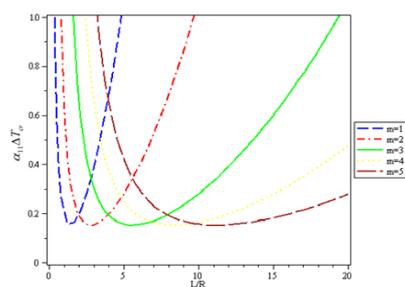


Figure 9. Changes of  $\alpha_{11}\Delta T_{cr}$  in terms L/R for  $f=10\%$  and fibers from type of armchair (6,6) for different longitudinal half waves and fiber angle of  $45^\circ$  and simple support

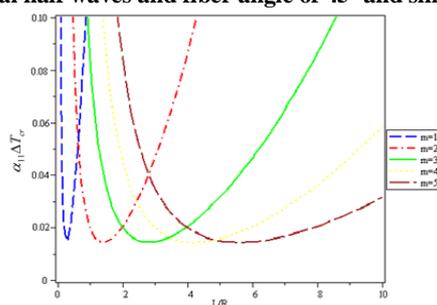


Figure 10. Changes of  $\alpha_{11}\Delta T_{cr}$  in terms L/R for  $f=10\%$  and fibers from type of armchair (6,6) for different longitudinal half waves and fiber angle of  $0^\circ$  and simple support

## CONCLUSION AND SUMMARIZING

- 1- presence of van der Waals force in interface of nanotube and polymer and decrease of radius of carbon nanotube increases membrane buckling strength
- 2- Openings emerged in the interface is the cause of reduction ineffective elasticity module of composite membrane.
- 3- By increase of fibers angle with respect to cylinder axis from  $0$  to  $45^\circ$  stability of nanocomposite membrane increases and then decreases.
- 4- Due to high elastic coefficient of carbon nanotube with respect to base, change in fibers volume percentage had no such impact in

fibers optimal angle but this change increased significantly stability amount.

- 5- At constant volume percentage with increase of length to radius, the critical buckling temperature decreases die to reduction of number of carbon nanotube at constant volume.
- 6- At constant length to radius ratio, with increase of fibers volume percentage the buckling critical temperature increases.

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